Exploring Square Roots and Irrational Numbers

1. Vocabulary Review
In a power, the \( n \) tells how many times a base is used as a factor. \( \text{exponent} \)

Evaluate the expression \( x^2 \) for each value of \( x \).

1. 2 \( \quad \)
2. 4 \( \quad \)
3. \(-2\) \( \quad \)
4. \(-4\) \( \quad \)

Finding Square Roots of Perfect Squares

Find the square roots of each number.

a. 36
b. 1

c. The symbol \( \sqrt{ \) means the square root of a number. In this book, \( \sqrt{ \) means the positive square root, unless stated otherwise. So \( \sqrt{9} \) means the positive square root of 9, or 3, and \( -\sqrt{9} \) means the opposite of the positive square root of 9, or \(-3\).

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<thead>
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<th>( n )</th>
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To estimate the square root of a number that is not a perfect square, use the square root of the nearest perfect square.

**EXAMPLE** Estimating a Square Root

Estimate the value of \( \sqrt{28} \) to the nearest integer.

\[
\begin{array}{ccc}
\sqrt{25} & \sqrt{28} & \sqrt{36} \\
5 & \ & 6
\end{array}
\]

Since 28 is closer to 25 than it is to 36, \( \sqrt{28} \) is closer to 5 than to 6. You can write \( \sqrt{28} \approx 5 \).

**Quick Check**

2. Estimate the value of \( \sqrt{38} \) to the nearest integer. 6

Finding a number’s square root is the inverse operation of finding the number’s square. So \( \sqrt{9} = 3 \).

**EXAMPLE** Application: Skydiving

The formula \( d = 16t^2 \) represents the approximate distance \( d \) in feet a skydiver falls in \( t \) seconds before opening the parachute. The formula assumes there is no air resistance. Find the time a skydiver takes to fall 816 feet before opening the parachute.

\[
\begin{align*}
\text{Given:} & \quad d = 816 \\
& \quad 16t^2 = 816 \\
& \quad t^2 = \frac{816}{16} \\
& \quad t^2 = 51 \\
& \quad \sqrt{51} \approx 7.1
\end{align*}
\]

The skydiver takes about 7.1 seconds to fall 816 feet.

**Quick Check**

3. Find the time a skydiver takes to fall each distance. Round to the nearest tenth of a second.

- a. 480 ft 5.5 s
- b. 625 ft 6.3 s

Irrational numbers are numbers that cannot be written in the form \( \frac{a}{b} \), where \( a \) is any integer and \( b \) is any nonzero integer. Rational and irrational numbers form the set of real numbers.
The decimal digits of irrational numbers do not terminate or repeat. The decimal digits of \( \sqrt{2} \) do not terminate or repeat, because \( \sqrt{2} \) is an irrational number. Irrational numbers can also include decimals that have a pattern in their digits, like \( 0.1010010001 \ldots \). For any integer \( n \) that is not a perfect square, \( \sqrt{n} \) is irrational.

**Classifying Real Numbers**

Identify each number as rational or irrational. Explain.

a. \(-9.3333\) Rational; the decimal repeats.

b. \(4\frac{2}{7}\) Rational; the ratio is \(\frac{43}{9}\).

c. \(\sqrt{90}\) Irrational; 90 is not a perfect square.

d. 6.3636636636666\ldots Irrational; the decimal does not terminate or repeat a group of digits.

**Closure**

- What is the square root of a given number? A number that when multiplied by itself is equal to the given number.
- Give several examples of irrational numbers. Sample: \(\sqrt{2}, \sqrt{3}, 1.3433443453444 \ldots \).
- Give several examples of rational numbers. Sample: \(\frac{3}{7}, \frac{\sqrt{5}}, 0.1010010001 \ldots\).

**Check Your Understanding**

**Vocabulary** Write all the possible names for each number. Choose from the terms at the right.

1. \(\sqrt{6}\) Irrational, real
2. \(\frac{1}{2}\) Rational, real
3. \(-0.5\) Irrational, real
4. \(25\) Rational, real, perfect square

Find the positive and negative square roots of each number.

5. 4, -2
6. \(\frac{1}{2}, -\frac{1}{2}\)
7. 10, -10
8. \(\frac{1}{10}, -\frac{1}{10}\)
1. Practice

Assignment Guide

Check Your Understanding

Go over Exercises 1–8 in class before assigning the Homework Exercises.

Homework Exercises

A Practice by Example 9–31
B Apply Your Skills 32–49
C Challenge 50

Test Prep and Mixed Review 51–56

Homework Quick Check

To check students’ understanding of key skills and concepts, go over Exercises 23, 27, 34, 36, and 47.

Differentiated Instruction Resources

3. Practice

Guided Problem Solving

The area of a square postage stamp is 49 in.². What is the side length of the stamp?

Estimate the value of each expression to the nearest integer.

A Find the square roots of each number.

1. \( \sqrt{9} \) 2. \( \sqrt{16} \) 3. \( \sqrt{25} \) 4. \( \sqrt{36} \) 5. \( \sqrt{49} \)

B Give an example of an irrational number.

32. Guided Problem Solving The area of a square postage stamp is 49 in.². What is the side length of the stamp?

Is each number rational or irrational? Explain. 26–31. See margin.

26. 0.3 27. \( \sqrt{40} \) 28. 0.606606606 . . .

29. \( \sqrt{144} \) 30. \( \sqrt{12} \) 31. 0.0203040506 . . .

33. Guided Problem Solving The area of a square postage stamp is 49 in.². What is the side length of the stamp?

37a. Yes; the sum of two irrational numbers is an irrational number.

b. Yes; the sum of two rational numbers is a rational number.

c. Yes; the sum of two prime numbers can be a composite number.

36. Writing in Math Explain how you can approximate \( \sqrt{30} \). See left.

37. The Closure Property states that a set of numbers is closed under a given operation if the result of the operation is in the same set of numbers. For example, the set of rational numbers is closed under addition, because the sum of any two rational numbers is a rational number. Is each set of numbers closed under addition? Explain.

a. even numbers  b. irrational numbers  c. prime numbers

37a–c. See left.

Online lesson quiz, PHSchool.com, Web Code: asa-0301 3-1 Exploring Square Roots and Irrational Numbers 109

26. Rational; the decimal terminates.

29. Rational; 144 is a perfect square.

27. Irrational; 40 is not a perfect square.

30. Irrational; 12 is not a perfect square.

28. Irrational; the decimal does not terminate or repeat.
4. Assess & Reteach

Lesson Quiz
1. Find the two square roots of 400. 20 and −20
2. Estimate \( \sqrt{34} \) to the nearest integer. 6
3. Using \( d = 16t^2 \), find how long it takes a skydiver to fall 676 ft from an airplane. 6.5 s
4. Is \( \sqrt[4]{25} \) rational or irrational? Explain. Rational; it can be written as \( \frac{5}{1} \)

Reteaching 3-1

Enrichment 3-1

Test Prep and Mixed Review

Multiple Choice
51. The area of a square is 150 square centimeters. Which best represents the side length of the square? B
   \( 11.7 \text{ cm} \) \( 12.2 \text{ cm} \) \( 2.9 \text{ cm} \) \( 13 \text{ cm} \)
52. The diameter of a human hair is about \( 1.7 \times 10^{-5} \) meters. Which of the following represents this number in standard notation? F
   \( 0.000017 \) \( 0.00017 \) \( 17,000 \) \( 170,000 \)
53. Which problem situation matches the equation \( 2x + 5 = 20 \)? C
   A. Jacob travels 5 more than twice as many miles to work as Carrie travels. If Carrie travels 20 miles to work, how many miles \( x \) does Jacob travel?
   B. Dana’s arm is 5 inches longer than Collin’s arm. If Collin’s arm is 20 inches long, what is twice the length \( x \) of Collin’s arm?
   C. Joel made a $20 phone call to Spain. The call cost $2 per minute plus a $5 connection fee. How many minutes \( x \) did the call last?
   D. Alondra invited 20 people to a party. Two people arrived late, and five people could not go. How many people \( x \) arrived on time for the party?

Write each number in scientific notation.
54. 18,000 \( 1.8 \times 10^4 \)
55. 6,038,000 \( 6.038 \times 10^6 \)
56. 49,700 \( 4.97 \times 10^4 \)

Alternative Assessment

Test Prep

Resources
For additional practice with a variety of test item formats:
- Test-Taking Strategies, p. 151
- Test Prep, p. 155
- Test-Taking Strategies with Transparencies

Each student in a pair writes an irrational number. Each partner decides which two whole numbers the other partner’s value falls between.

49. The student took the square root of 4 and added it to the square root of 9. You must add 4 + 9 first and then take the square root.